

Rotation

CS418 Computer Graphics

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Homogenous Transformation Matrices

- Scale

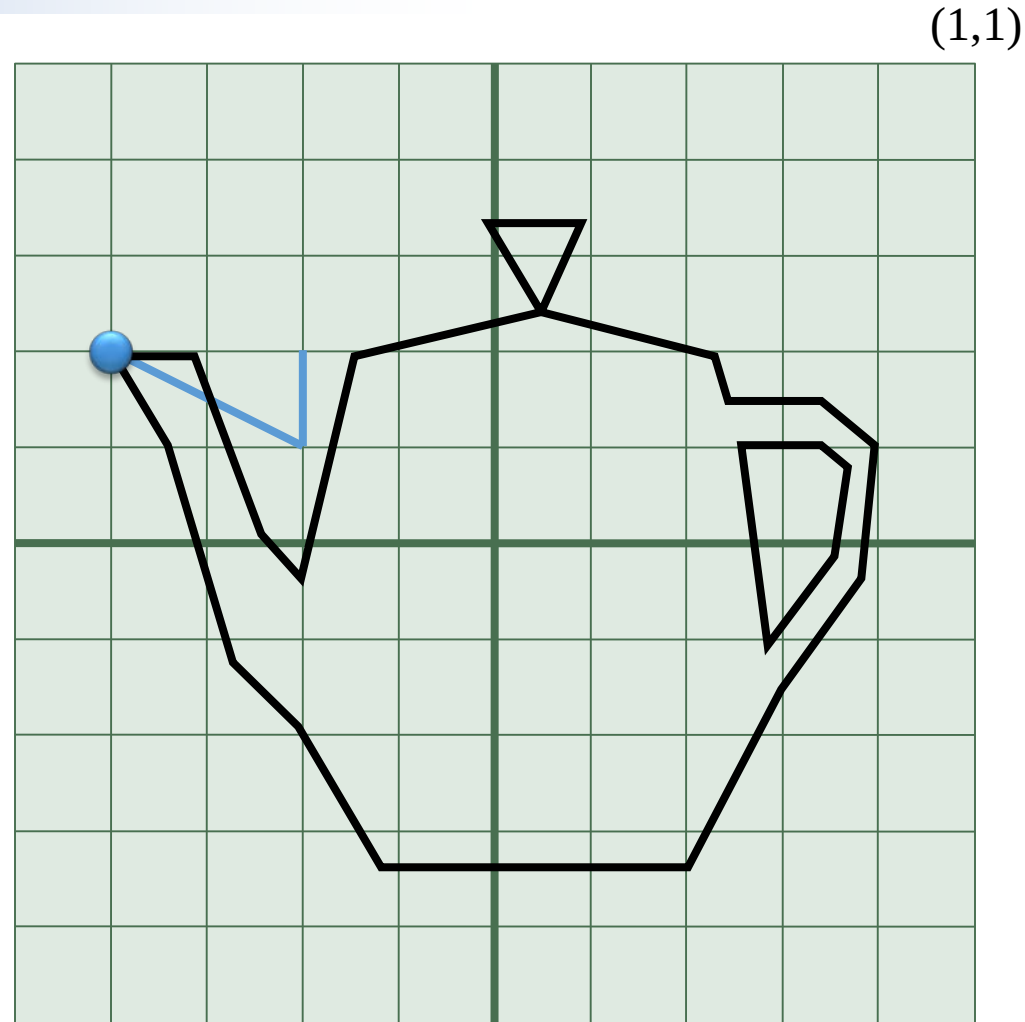
$$\begin{bmatrix} s & & \\ & s & \\ & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} sx \\ sy \\ 1 \end{bmatrix}$$

- Translate

$$\begin{bmatrix} 1 & & a \\ & 1 & b \\ & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + a \\ y + b \\ 1 \end{bmatrix}$$

- Combination

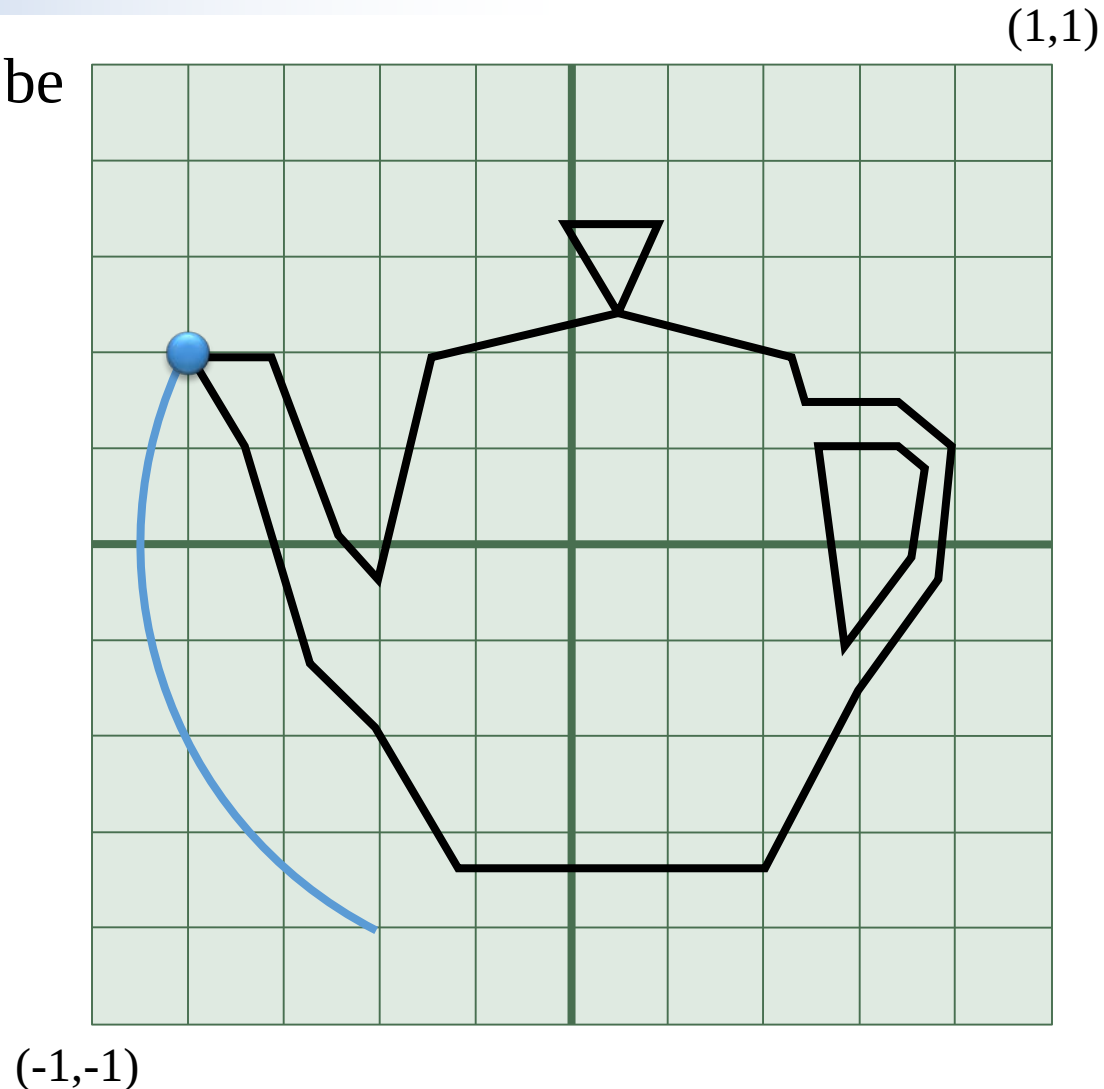
$$\begin{bmatrix} 1 & & a \\ & 1 & b \\ & & 1 \end{bmatrix} \begin{bmatrix} s & & \\ & s & \\ & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} sx + a \\ sy + b \\ 1 \end{bmatrix} \quad (-1, -1)$$



Rotation Matrix

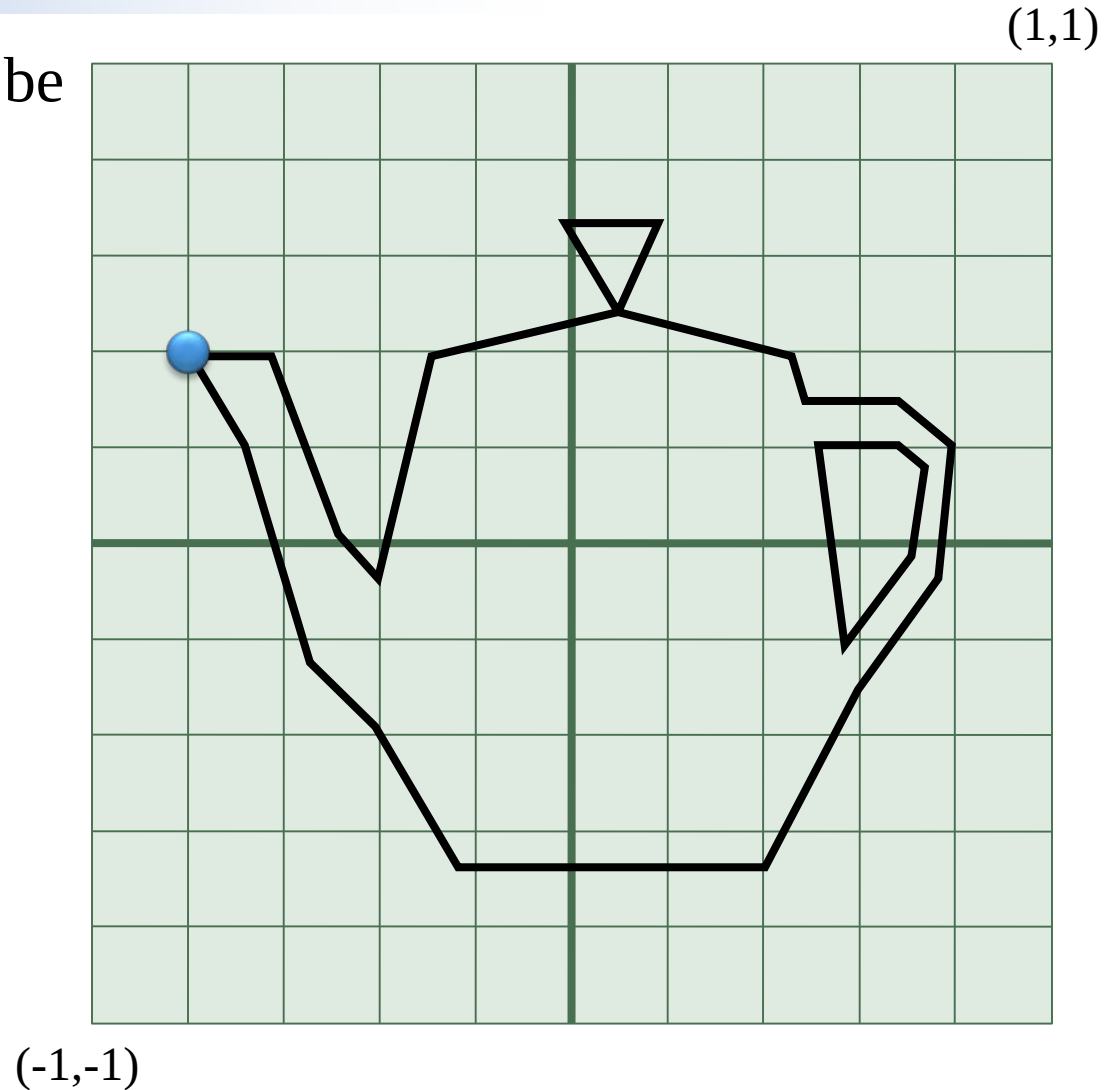
- What transformation matrix can be used to rotate an object?

$$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} -0.8 \\ 0.4 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.4 \\ -0.8 \\ 1 \end{bmatrix}$$



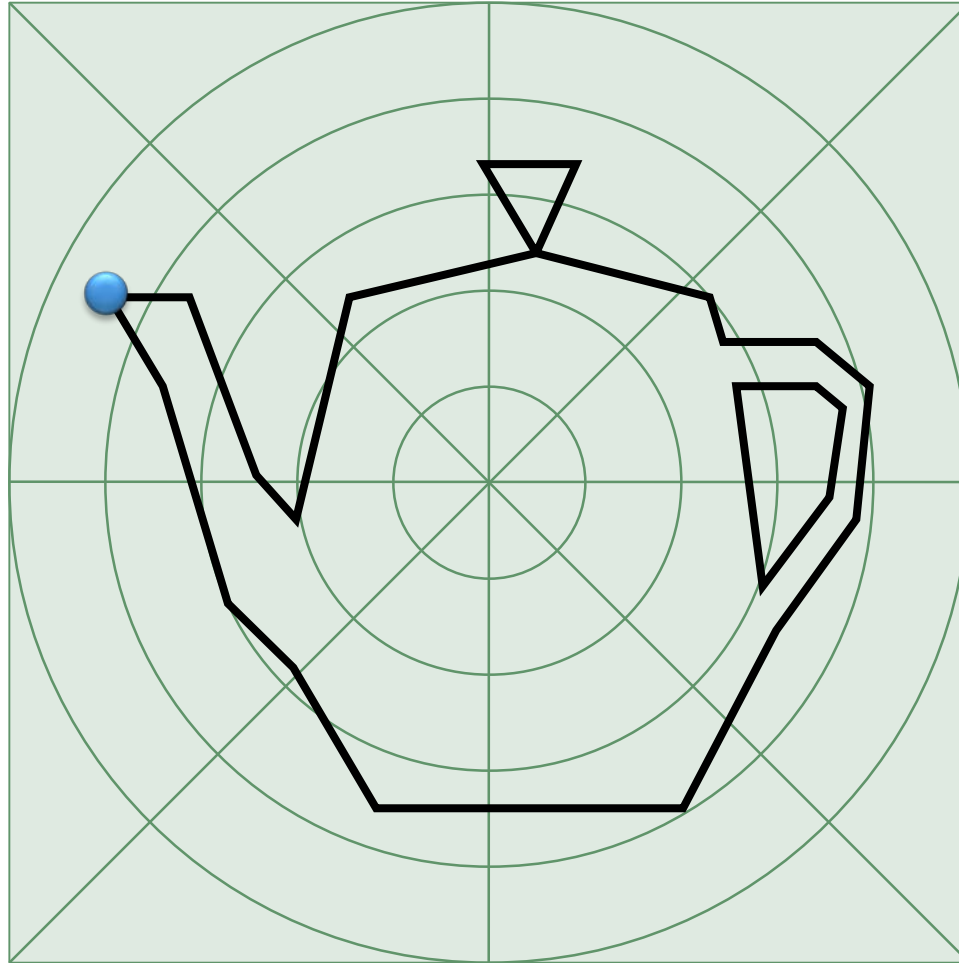
Rotation Matrix

- What transformation matrix can be used to rotate an object?
- Cartesian $\rightarrow$ polar coordinates



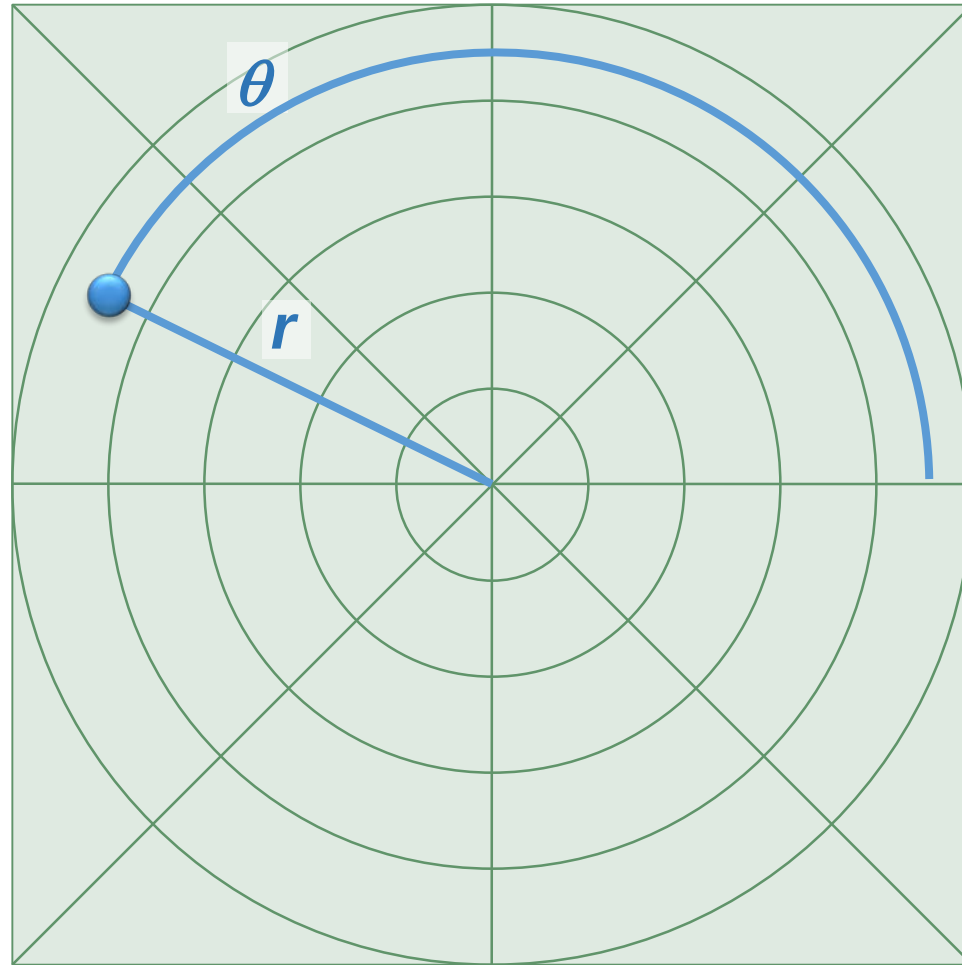
Rotation Matrix

- What transformation matrix can be used to rotate an object?
- Cartesian $\rightarrow$ polar coordinates



Rotation Matrix

- What transformation matrix can be used to rotate an object?
- Cartesian $\rightarrow$ polar coordinates
 $x = r \cos \theta, y = r \sin \theta$



Rotation Matrix

- What transformation matrix can be used to rotate an object?

- Cartesian $\rightarrow$ polar coordinates

$$x = r \cos \theta, y = r \sin \theta$$

- Rotate to (x', y')

$$x' = r \cos(\theta + \phi), y' = r \sin(\theta + \phi)$$

- Recall trig identities

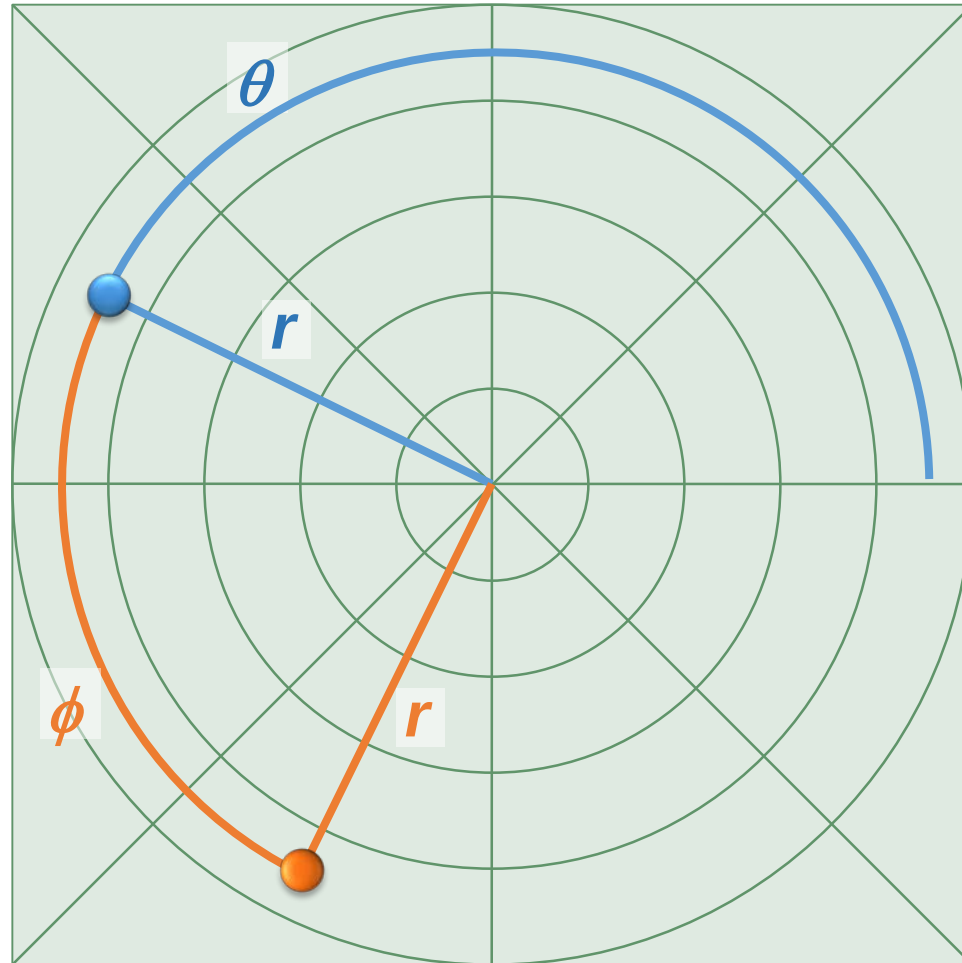
$$x' = r (\cos \theta \cos \phi - \sin \theta \sin \phi)$$

$$y' = r (\sin \theta \cos \phi + \cos \theta \sin \phi)$$

- Rearrange terms

$$x' = \cos \phi (r \cos \theta) - \sin \phi (r \sin \theta)$$

$$y' = (r \cos \theta) \sin \phi + (r \sin \theta) \cos \phi$$



Rotation Matrix

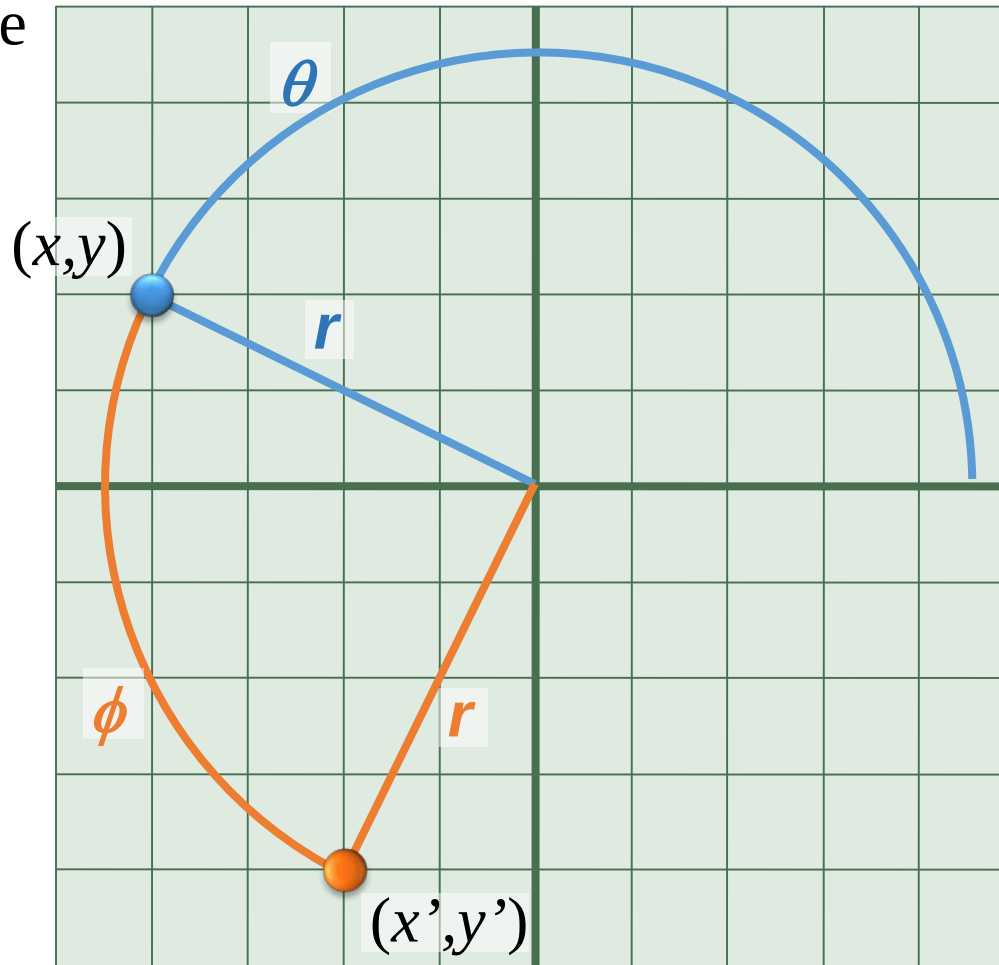
- What transformation matrix can be used to rotate an object?

- Cartesian $\rightarrow$ polar coordinates

$$x = r \cos \theta, y = r \sin \theta$$

$$\begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\begin{aligned} x' &= \cos \phi (x) - \sin \phi (y) \\ y' &= (x) \sin \phi + (y) \cos \phi \end{aligned}$$



Where Does the Minus Sign Go?

$$\begin{bmatrix} c & -s & \\ s & c & \\ & & 1 \end{bmatrix} \text{ or } \begin{bmatrix} c & s & \\ -s & c & \\ & & 1 \end{bmatrix}$$

- Protip: set rotation angle to 90°

$$\begin{bmatrix} 0 & -1 & \\ 1 & 0 & \\ & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

